V-MAD III
Tercer Encuentro en Aplicaciones de la Matemática
Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso

Exact boundary conditions for numerical treatment of semi-infinite domains

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INGMAT S.A.

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Framework

- Mathematical Modelling and Partial Differential Equations
- Scientific Computing and engineering applications
- Numerical solution of problems in unbounded domains
- Exact boundary conditions for semi-infinite domains

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2. Concept of artificial boundary conditions
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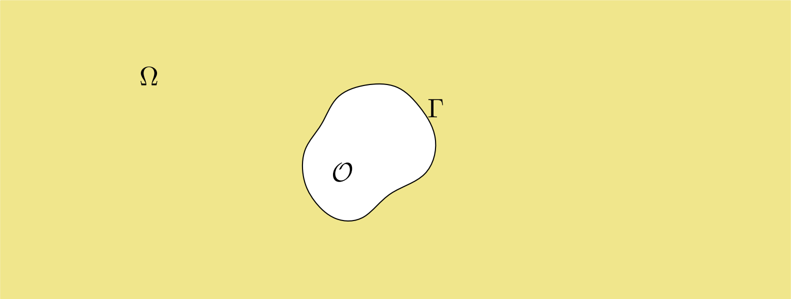
4. Exact boundary conditions for semi-infinite domains

Exterior domains



Some applications:

- Scattering by an obstacle
- Fluid flow around an obstacle

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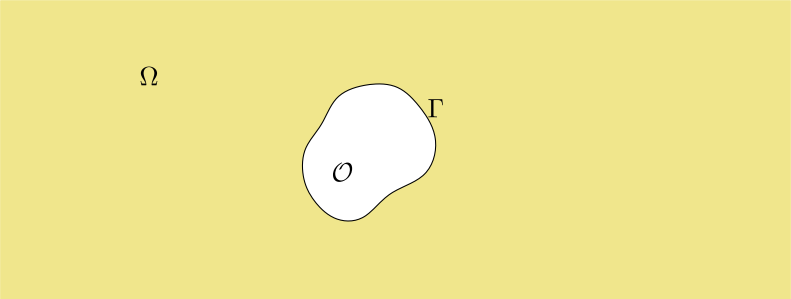

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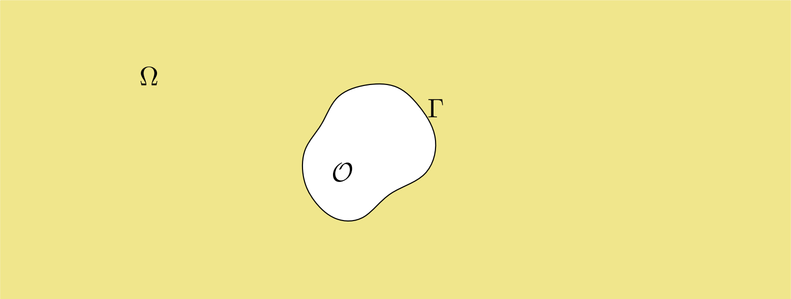

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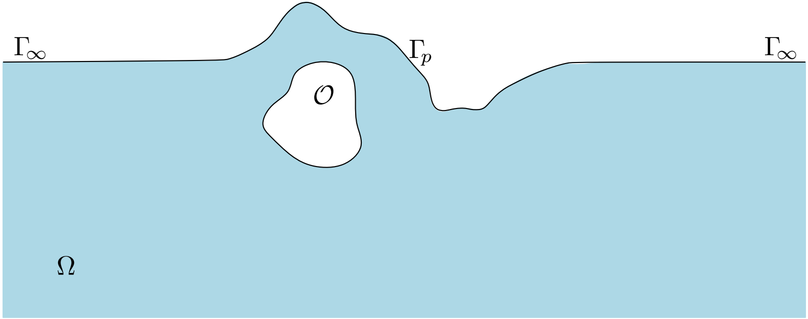

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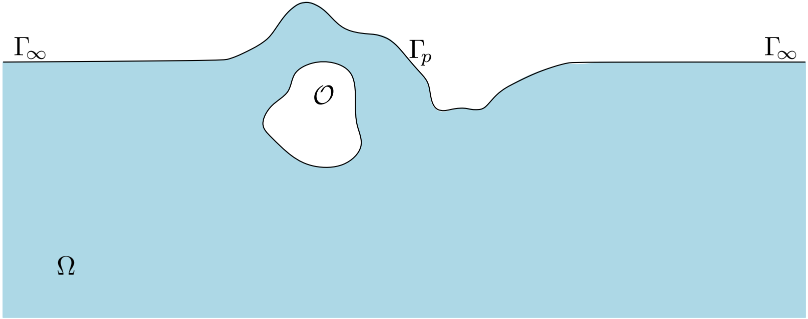

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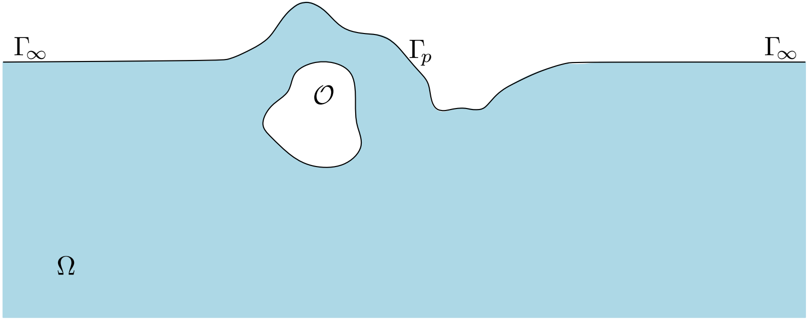

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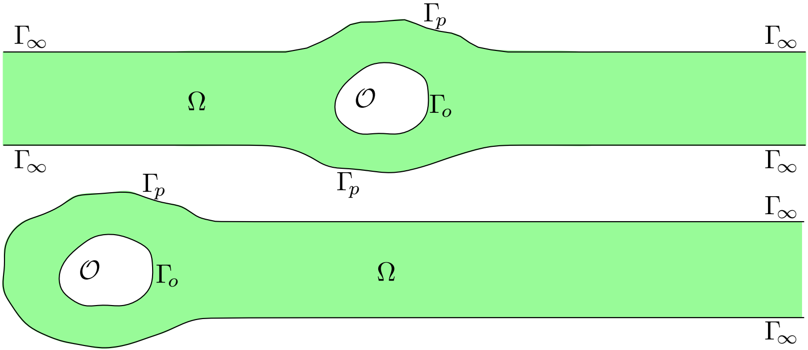

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- Wave guides
- Flow through ducts

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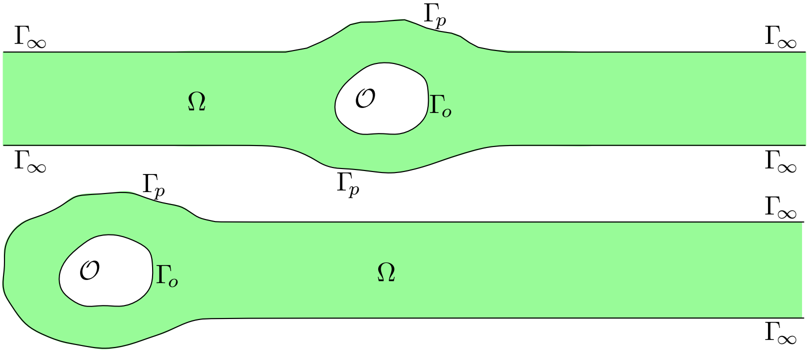

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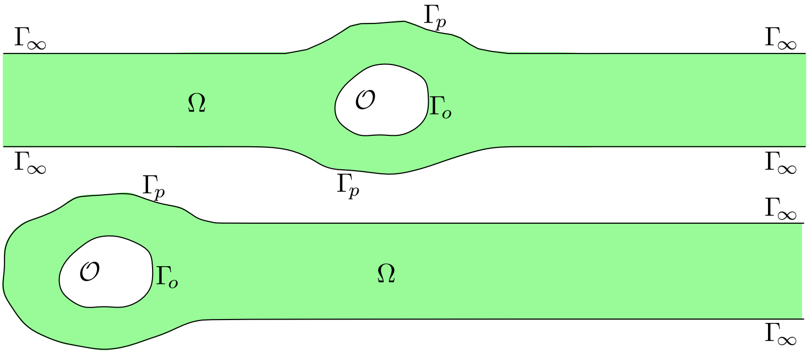

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Main methods for unbounded domain problems

- Boundary integral / Boundary element methods
- Infinite elements
- Absorbing layers
- Artificial boundary conditions

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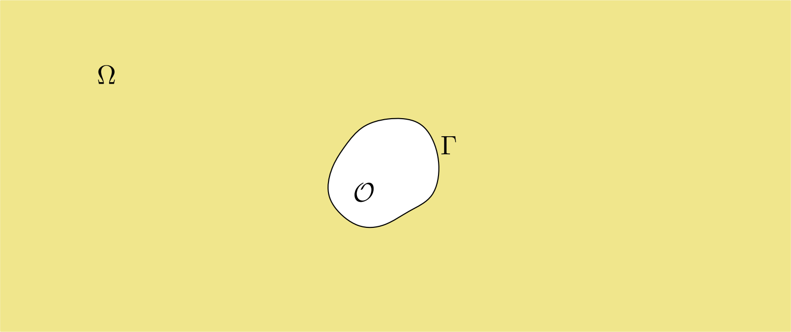

- Involved domain Ω has infinite size
- It cannot be stored in a computer

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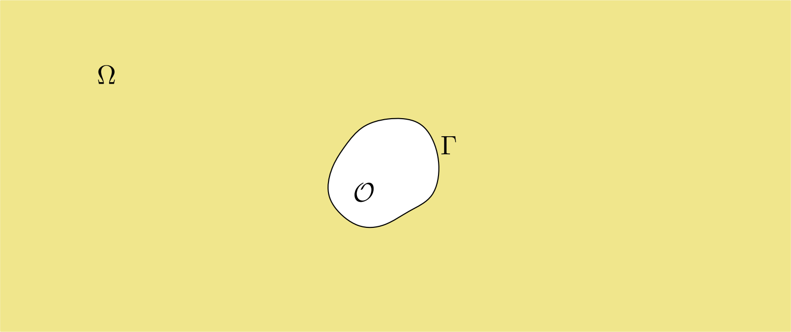

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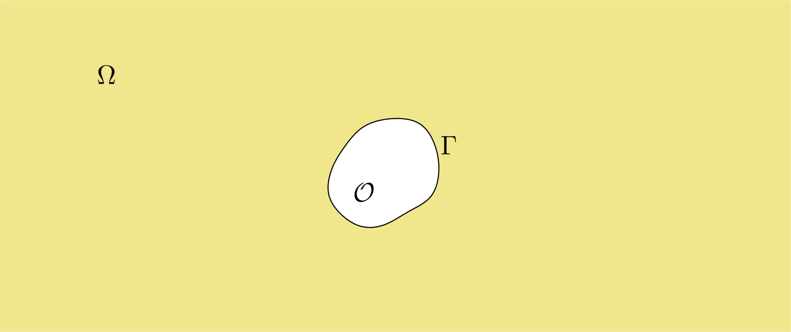

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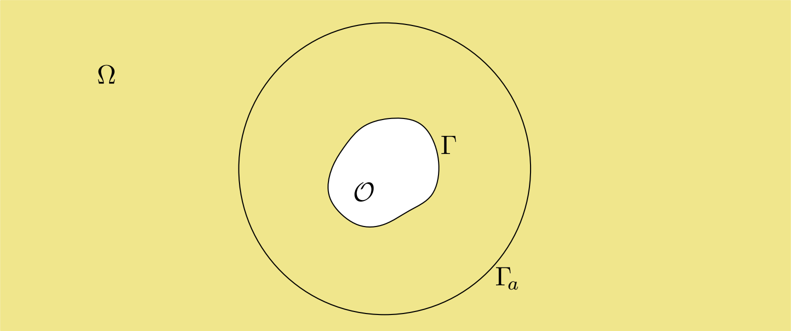
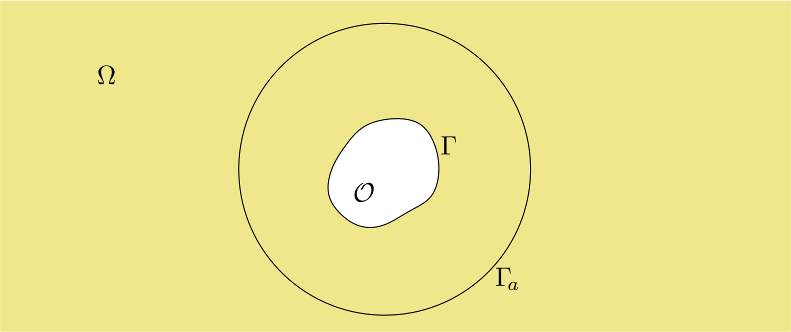


- Original domain Ω divided into two domains:
  - An external residual domain Ω_e (unbounded)
  - An internal computational domain Ω_i (bounded)

Artificial boundary



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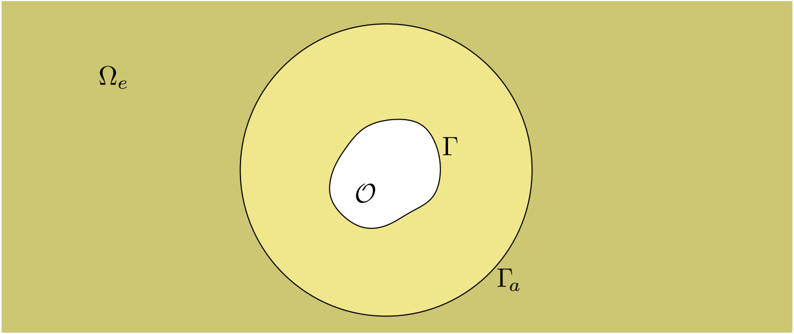

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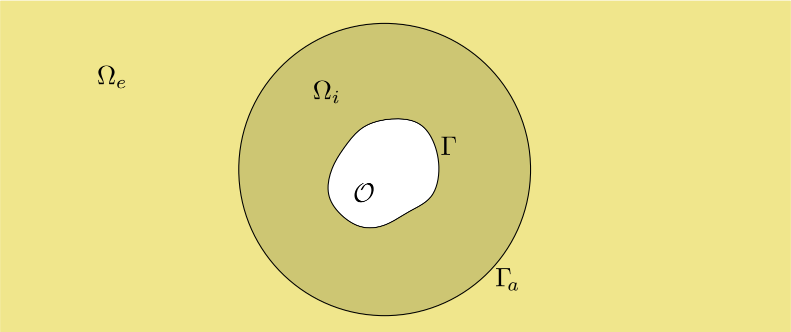

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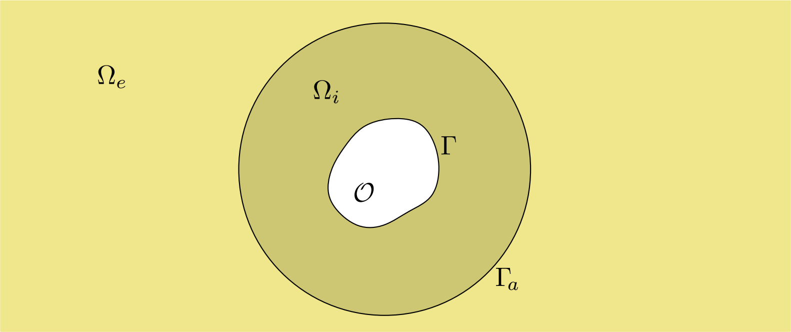

- Numerical calculations are restricted to Ω_i
- A suitable boundary condition must be prescribed on Γ_a

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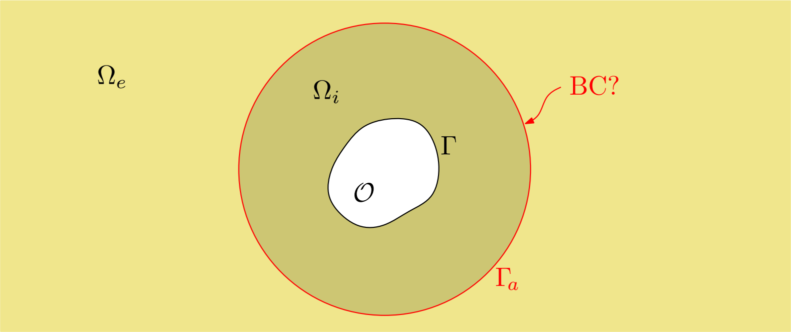

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Artificial boundary conditions

- Any such boundary condition, suitable in a sense to be defined, is called an artificial boundary condition (ABC) (or nonreflecting / absorbing BC in wave problems)
- Desirable properties of an ABC
  - To ensure the solvability of the problem on Ω_i
  - The solution computed on Ω_i is close to the solution of the original problem on Ω when restricted to Ω_i
- If both solutions coincide exactly, the ABC is called exact
- Otherwise, the ABC is called approximate

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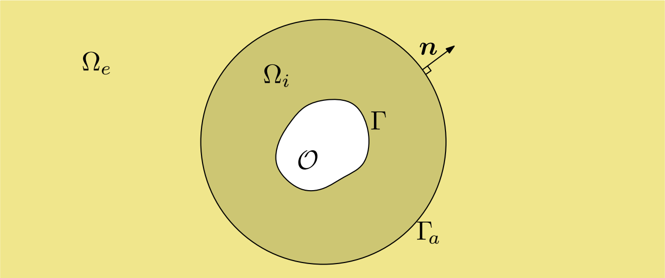
Exact boundary condition via a DtN map



- Solve analytically by separation of variables a boundary value problem on Ω_e with a Dirichlet data u on Γ_a
- Compute the normal derivative on Γ_a as a function of u
- This procedure defines a linear operator $\mathcal{T}$ called the Dirichlet-to-Neumann (DtN) map

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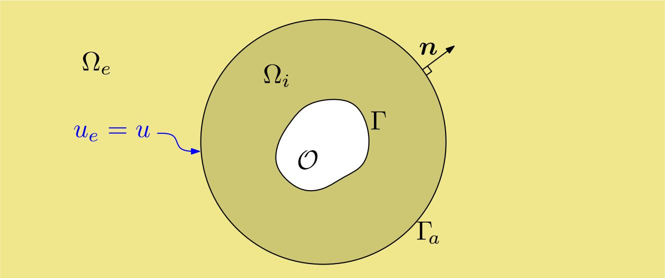

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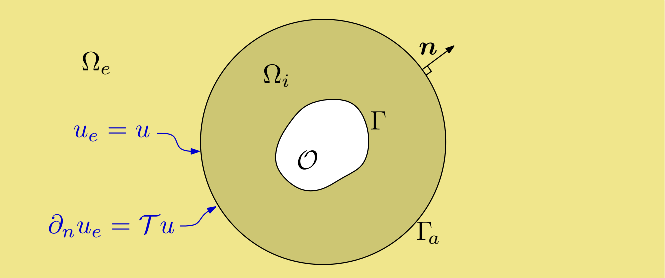

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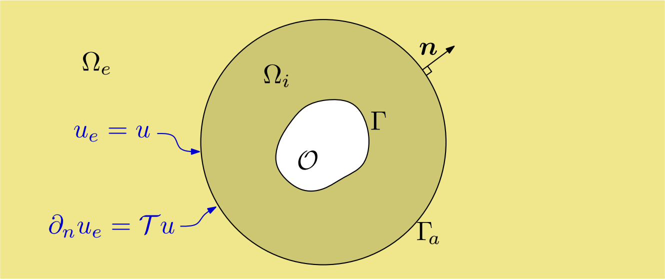

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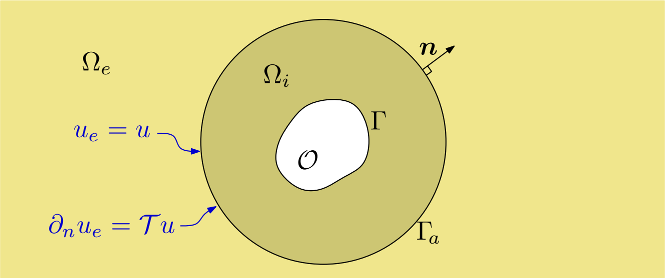

- The DtN map provides an exact boundary condition on Γ_a, which is non-local in the spatial variable
- The resulting boundary value problem on Ω_i is well-posed
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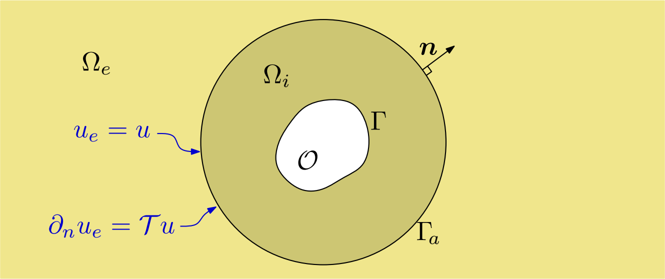

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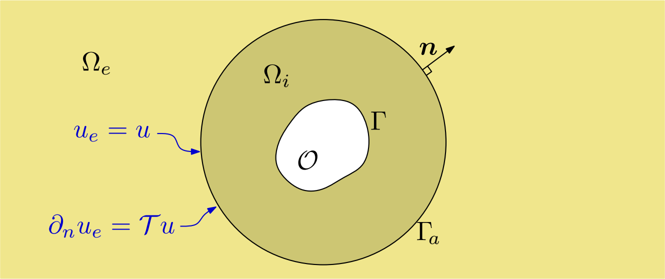

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Some examples of exact boundary conditions

Laplace in 2D: $\triangle u = 0$ in Ω, $\Gamma_a = S_1(0,R)$

$$\mathcal{T}u(\theta) = -\sum_{n=1}^{\infty} \int_{\Gamma_a} m_n(\theta,\theta') u(\theta')\,\mathrm{d}s(\theta')$$

$$m_n(\theta,\theta') = \frac{n}{\pi R^2} \cos n(\theta - \theta')$$

cf. Givoli & Keller (1989) *A finite element method for large domains*

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Helmholtz in 2D, $\triangle u + k^2 u = 0$ in Ω, $\Gamma_a = S_1(0,R)$

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cf. Keller & Givoli (1989) *Exact non-reflecting boundary conditions*

Some examples of exact boundary conditions

Laplace in 3D, $\triangle u = 0$ in Ω, $\Gamma_a = S_2(0, R)$

$$\mathcal{T}u(\theta,\phi) = -\sum_{n=0}^{\infty}\int_{\Gamma_a} m_n(\theta,\phi,\theta',\phi')u(\theta',\phi')\,\mathrm{d}s(\theta',\phi')$$

$$m_n(\theta,\phi,\theta',\phi') = \sum_{j=0}^{n}\beta_{jn}P_n^j(\cos\phi)P_n^j(\cos\phi')\cos j(\theta-\theta')$$

$$\beta_{0n} = \frac{(2n+1)(n+1)}{4\pi R^3}, \qquad \beta_{jn} = \frac{(2n+1)(n+1)(n-j)!}{2\pi R^3(n+j)!}$$

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Further related references

Givoli & Keller (1990) *Non-reflecting boundary conditions for elastic waves*

Han & Wu (1992) *The approximation of the exact boundary conditions at an artificial boundary for linear elastic equations and its application*

Grote & Keller (1995) *Exact non-reflecting boundary conditions for the time-dependent wave equation*

Harari & Shohet (1998) *On non-reflecting boundary conditions in unbounded elastic solids*

Grote & Keller (1998) *Non-reflecting boundary conditions for Maxwell's equations*

Gächter & Grote (2003) *Dirichlet-to-Neumann map for three-dimensional elastic waves*

Coupling FEM with DtN

The weak formulation of the problem on Ω_i looks like:

$$(P)\begin{cases} \text{Find } u \in V \text{ such that} \\ a(u,v)+b(u,v)=\ell(v) \quad \forall v \in V \end{cases}$$

with V, a and ℓ as usual in the context of Lax-Milgram lemma

The bilinear form $b(u,v)$ is related to the DtN map through

$$b(u,v) = -\int_{\Gamma_a} v(\theta)\,\mathcal{T}u(\theta)\,\mathrm{d}s(\theta)$$

If (P) is discretised using a FEM scheme with shape functions $\{\psi_j\}$, it is necessary to compute the matrix $B_{ij} = b(\psi_j, \psi_i)$

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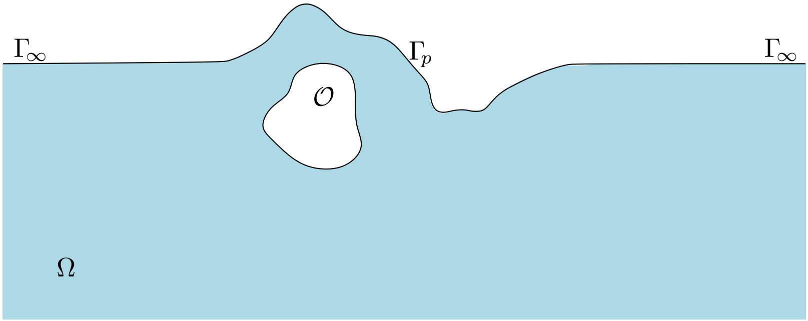
Case of a semi-infinite domain



- Artificial boundary delimited by the infinite boundary Γ_∞
- The artificial boundary condition to be prescribed on Γ_a must agree with that on Γ_∞

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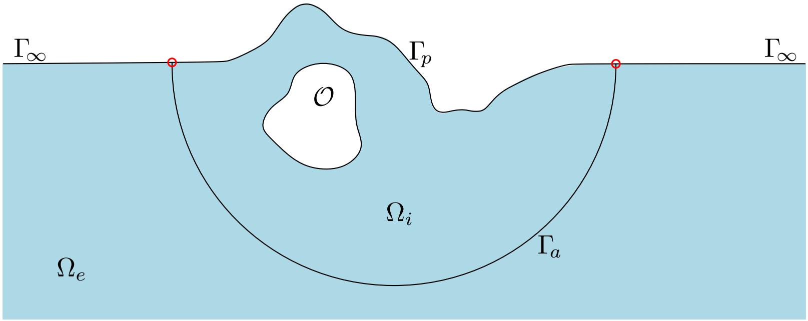

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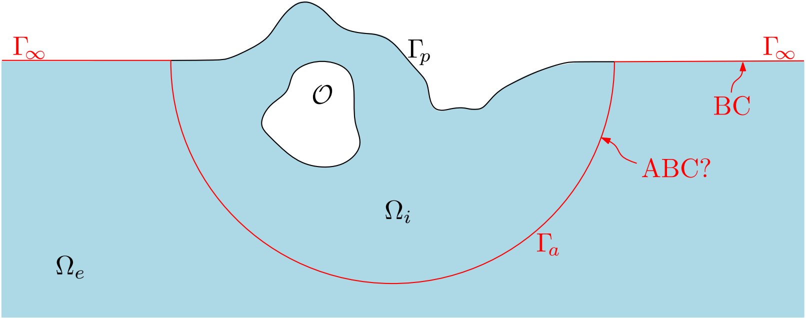

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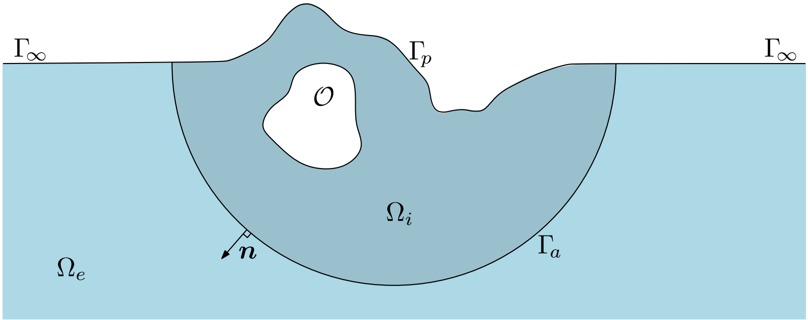
Adaptation of exact boundary conditions



- Obtain the DtN map $\mathcal{T}$ in function of the Dirichlet data u
- Use $\mathcal{T}$ to prescribe exact boundary conditions on Γ_a
- Solve numerically the obtained well-posed problem on Ω_i

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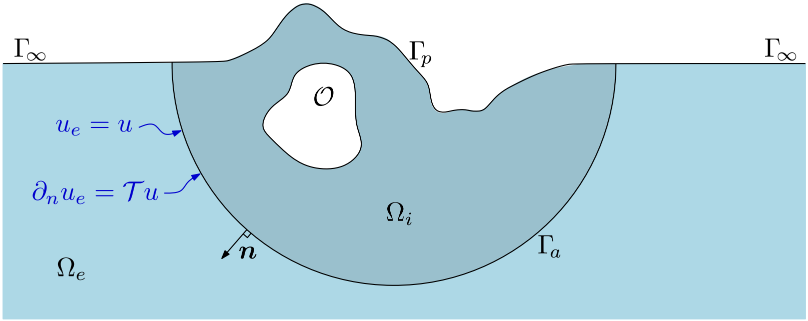

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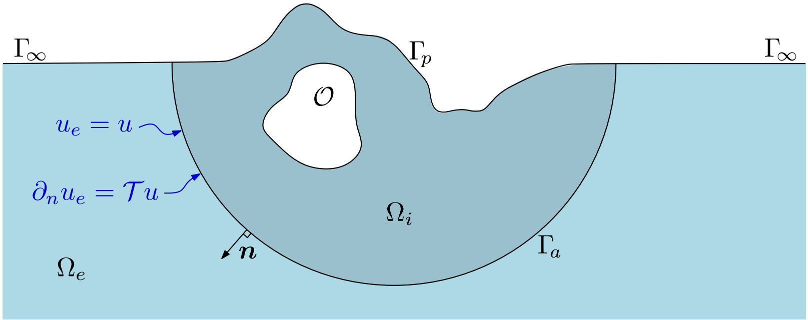

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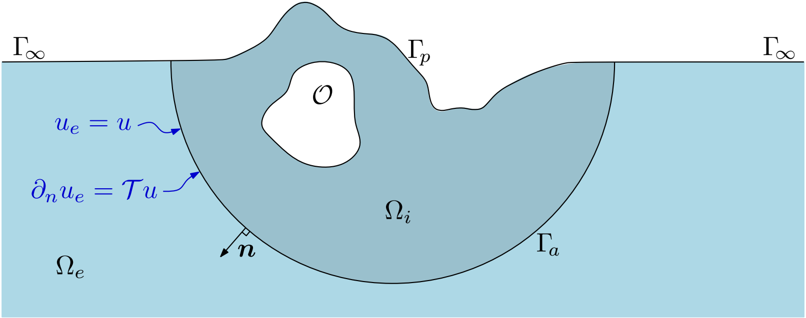

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Some examples of exact boundary conditions

Helmholtz in 2D:
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$$\mathcal{T}u(\theta) = \sum_{n=0}^{\infty} \int_{\Gamma_a} m_n(\theta, \theta') u(\theta') \, \mathrm{d}s(\theta')$$

$$m_n(\theta, \theta') = -\frac{2k}{\pi R} \frac{H_n^{(1)\prime}(kR)}{H_n^{(1)}(kR)} \cos n\theta \cos n\theta'$$

cf. Givoli & Vigdergauz (1993) *Artificial boundary conditions for 2D problems in geophysics*

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Elastostatics in 2D:

$$\partial_j \sigma_{ij}(\boldsymbol{u}) = \mathbf{0} \text{ in } \Omega, \quad \sigma_{ij}(\boldsymbol{u}) n_j = \mathbf{0} \text{ on } \Gamma_\infty, \quad \Gamma_a = S_1^-(0, R)$$

The method of separation of variables fails in solving the Dirichlet boundary value problem on Ω_e

A solution method based on complex analysis is employed

The solution involves an infinite set of coefficients given as a solution of an infinite system of linear equations

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Some examples of exact boundary conditions

Elastostatics in 2D:

$\partial_j \sigma_{ij}(\boldsymbol{u}) = \mathbf{0}$ in Ω, $\quad \sigma_{ij}(\boldsymbol{u}) n_j = \mathbf{0}$ on Γ_∞, $\quad \Gamma_a = S_1^-(0, R)$

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The right-hand side of the infinite system depends on the Dirichlet data $\boldsymbol{u}$

For each $\boldsymbol{u}$ the infinite system is truncated and an approximation of $\mathcal{T}\boldsymbol{u}$ is computed

The DtN map this time is not given in closed-form

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Coupling FEM with a DtN not given in closed-form

The DtN map is present in the weak formulation through the bilinear form $b(u,v)$

$$b(u,v) = -\int_{\Gamma_a} v(\theta)\,\mathcal{T}u(\theta)\,\mathrm{d}s(\theta)$$

Discretising using a FEM scheme with shape functions $\{\psi_j\}$, it is necessary to compute the matrix

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In practice, the exact boundary condition is approximated

Its coupling with the FEM scheme is more difficult to implement and more expensive computationally

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Current research project

Stresses induced in rock by mining activity

Joint work by

Eduardo Godoy R. (Research Engineer, INGMAT S.A.)
Valeria Boccardo S. (Ph.D. Student, PUC)
Mario Durán T. (Associated Professor, PUC)

- Elastostatics in a semi-infinite domain with axisymmetry
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- Exact boundary conditions based on the DtN map allows to treat numerically problems involving unbounded domains
- Closed-forms of the DtN map exist for exterior domains in many cases and for semi-infinite domains in just a few cases
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Thanks for your attention!